

Entropy of (1+1)-dimensional charged black hole to all orders in the Planck length

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Abstract

We study the statistical entropy of a scalar field on the (1+1)-dimensional Maxwell-dilaton background without an artificial cutoff considering corrections to all orders in the Planck length from a generalized uncertainty principle (GUP) on the quantum state density. In contrast to the previous results of the higher dimensional cases having adjustable parameter, we obtain an unadjustable entropy due to the independence of the minimal length while this entropy is proportional to the Bekenstein-Hawking entropy.

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I. INTRODUCTION

Since 't Hooft statistically obtained the Bekenstein-Hawking entropy [1] of a scalar field outside the horizon of the Schwarzschild black hole by introducing an artificial brick wall cutoff in order to remove the ultraviolet divergence near the horizon [2] and using the leading order Wenzel-Kramers-Brillouin (WKB) approximation, this method has been widely used to study the statistical property of bosonic and fermionic fields in various black holes [3, 4, 5]. However, there are some drawbacks in this brick wall model such as little mass approximation, neglecting logarithmic term and taking the infrared term L^3 as a contribution of the vacuum surrounding a black hole. Solving these problems, an improved brick-wall method (IBWM) has been introduced by taking the thin-layer outside the event horizon of a black hole as the integral region [6]. As a result, this IBWM has been solved these drawbacks except the artificial cutoffs.

Recently, in Refs. [7, 8, 9], the authors calculated the entropy of black holes to leading order in the Planck length by using the newly modified equation of states of density motivated by a GUP [10, 11], which drastically solves the ultraviolet divergences of the just vicinity near the horizon replacing the brick wall cutoff with the minimal length. In particular, we have studied the entropy of a scalar field on the (1+1)-dimensional charged black hole background up to leading order by using the GUP [12]. However, Yoon et. al. have very recently pointed out that since the minimal length $\sqrt{\lambda}$ is actually related to the brick wall cutoff ϵ , the integral about r in the range of the near horizon should be carefully treated for a convergent entropy [13].

It is also well-known that the deformed Heisenberg algebra [10] leads to a GUP showing the existence of the minimal length [14, 15, 16, 17], which originates from the quantum fluctuation of the gravitational field [18, 19]. On the other hand, quantum gravity phenomenology has been tackled with effective models based on GUPs and/or modified dispersion relations [20, 21] containing the minimal length as a natural ultraviolet cutoff [22]. The essence of ultraviolet finiteness of the Feynman propagator, which displays an exponential ultraviolet cutoff of the form of $\exp(-\lambda p^2)$, can be also captured by a nonlinear relation $p = f(k)$, where p and k are the momentum and the wave vector of a particle, respectively, generalizing the

commutation relation between operators \hat{x} and \hat{p} to

$$[\hat{x}, \hat{p}] = i\hbar \frac{\partial p}{\partial k} \Leftrightarrow \Delta x \Delta p \geq \frac{\hbar}{2} \left| \left\langle \frac{\partial p}{\partial k} \right\rangle \right| \quad (1)$$

at quantum mechanical level [21]. Recently, Nouicer has extended the calculation of entropy to all orders in the Planck length for the Randall-Sundrum brane case by arguing that a GUP up to leading order correction in the Planck length is not enough [14] because the wave vector k does not satisfy the asymptotic property in the modified dispersion relation [21]. After this work, we have obtained the entropy to all orders for the 4D Schwarzschild case by carefully considering the integral about r in the range of the near horizon [23]. Very recently, we have numerically calculated the entropy to all order corrections of a scalar field on the (2+1)-dimensional DS black hole background introducing the incomplete Γ -functions [24].

In this paper, we study the entropy to all order corrections in the Planck length of a scalar field on the charged black hole background in the lowest (1+1) dimensions carefully considering the integral about r in the range $(r_+, r_+ + \epsilon)$ near the horizon. As expected, this study of the 2D case is non-trivial in contrast to the 4D Schwarzschild case [23] because we should introduce the incomplete Γ -function and carry out numerical calculation as the 3D de Sitter case [24]. By using the novel equation of states of density [14] motivated by the GUP, we calculate the quantum entropy of a massive scalar field on the (1+1)-dimensional charged black hole background without any artificial cutoff and little mass approximation while satisfying the asymptotic property of the wave vector k in the modified dispersion relation. In contrast to the previous results of the higher dimensional cases having adjustable parameter α [23, 24], we obtain an unadjustable entropy while this entropy is proportional to the Bekenstein-Hawking entropy. From now on, we take the units as $G = \hbar = c = k_B \equiv 1$.

Let us now start with the two-dimensional Maxwell-dilaton action induced by the low energy heterotic string theory [25, 26], which is given by

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} [R + 4(\nabla\phi)^2 + 4\Lambda^2 - \frac{1}{4}F^2], \quad (2)$$

where ϕ is a dilaton field, Λ^2 a cosmological constant, and F a Maxwell field tensor. In the Schwarzschild gauge, the metric and field tensors are assumed to be

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2, \quad (3)$$

$$F_{rt} = F_{rt}(r). \quad (4)$$

Simple static solutions with suitable boundary conditions are known as follows:

$$\phi(r) = \frac{1}{4} \ln 2 - \Lambda r, \quad (5)$$

$$F_{rt}(r) = \sqrt{2} Q e^{-2\Lambda r}, \quad (6)$$

$$f(r) = 1 - \frac{M}{\Lambda} e^{-2\Lambda r} + \frac{Q^2}{4\Lambda^2} e^{-4\Lambda r}. \quad (7)$$

There are two coordinate singularities r_{\pm} which correspond to the positions of the outer event horizon and the inner Cauchy horizon:

$$r_{\pm} = \frac{1}{2\Lambda} \ln \left[\frac{M}{2\Lambda} \pm \sqrt{\left(\frac{M}{2\Lambda}\right)^2 - \left(\frac{Q}{2\Lambda}\right)^2} \right], \quad (8)$$

where the Cosmic censorship leads to the condition $M \geq Q$. The Hawking temperature is given by the surface gravity [27] as $T_H = \frac{\Lambda}{2\pi} [1 - e^{-2\Lambda(r_+ - r_-)}]$.

In this Maxwell-dilaton background, let us consider a scalar field with mass m under the background (5) and (6), which satisfies the Klein-Gordon equation $(\square - m^2)\Phi = 0$. Substituting the wave function $\Phi(r, \theta, t) = e^{-i\omega t} R(r, \theta)$, we find that the Klein-Gordon equation becomes

$$\frac{d^2 R}{dr^2} + \frac{1}{f} \frac{df}{dr} \frac{dR}{dr} + \frac{1}{f} \left(\frac{\omega^2}{f} - m^2 \right) R = 0. \quad (9)$$

By using the leading order WKB approximation [2] with $R \sim \exp[iS(r, \theta)]$, we have

$$p_r^2 = \frac{1}{f} \left(\frac{\omega^2}{f} - m^2 \right), \quad (10)$$

where $p_r = dS/dr$. On the other hand, we also have the square module momentum

$$p^2 = p_r p^r = g^{rr} p_r^2 = f p_r^2 = \frac{\omega^2}{f} - m^2 \quad (11)$$

with the condition $\omega \geq m\sqrt{f}$.

Considering the modified dispersion relation (1), the usual momentum measure dp_r is deformed to be $dp_r \frac{\partial k_r}{\partial p_r}$. Then, according to the Refs. [16, 17], we have

$$\frac{\partial p_r}{\partial k_r} = e^{\lambda p^2}, \quad (12)$$

where λ is a constant of order one in the Planck length units having the dimension of 1/momentum squared. Note that in the limit of $\lambda \rightarrow 0$, we recover the usual Heisenberg commutation relation. Moreover, this particular type of the nonlinear dispersion relation is

invertible and satisfies the requirement that for small energies than the cutoff the usual dispersion relation is recovered, while for large energies the wave vector asymptotically reaches the cutoff. These criteria [21] for selecting a proper Ansatz among different alternatives [28] have been also investigated in various context in phenomenological consequences [10]. Then, the deformed algebra, which is given by $[X, P] = i e^{\lambda P^2}$ with the representations $X \equiv i e^{\lambda P^2} \partial_p$ and $P \equiv p$ of the position and momentum operators, respectively, leads to the generalized uncertainty relation including all order corrections as

$$\Delta X \Delta P \geq \frac{1}{2} \langle e^{\lambda P^2} \rangle \geq \frac{1}{2} e^{\lambda((\Delta P)^2 + \langle P \rangle^2)}. \quad (13)$$

Note that $\langle P^{2n} \rangle \geq \langle P^2 \rangle^n$ and $(\Delta P)^2 = \langle P^2 \rangle - \langle P \rangle^2$.

Next, in order to investigate the quantum implications of this deformed algebra, let us solve the above relation (13) for ΔP that is satisfied with the equality sign. Then, the momentum uncertainty is simply given by

$$\Delta P = \frac{e^{\lambda \langle P \rangle^2}}{2 \Delta X} e^{\lambda (\Delta P)^2}. \quad (14)$$

On the other hand, if we define $W(\xi) \equiv -2\lambda (\Delta P)^2$ with $\xi = \frac{\lambda}{2(\Delta X)^2} e^{-2\lambda \langle P \rangle^2}$, we obtain the relation $W(\xi) e^{W(\xi)} = \xi$, which is just the definition of the Lambert function [29]. In order to have a real physical solution for ΔP , the argument of the Lambert function is required to satisfy $\xi \geq -1/e$, which naturally leads to the position uncertainty as $\Delta X \geq \sqrt{e\lambda/2} e^{\lambda \langle P \rangle^2} \equiv \Delta X_{\min}$. Here, ΔX_{\min} is a minimal uncertainty in position. Moreover, this minimal length intrinsically derived for physical states with $\langle P \rangle = 0$ is given by $\Delta X_0^A = \sqrt{e\lambda/2}$, which is the absolutely smallest uncertainty in position. In fact, this minimal length plays a role of the brick wall cutoff effectively giving the thickness of the thin-layer near the horizon [7, 8, 9, 12]. Note that the minimal length up to the leading order in a series expansion of Eq. (14) around $\langle P \rangle = 0$ is given by $\Delta X_0^L = \sqrt{\lambda} < \Delta X_0^A$, where the superscripts denote the leading order (L) and all orders (A), respectively. However, only this leading order correction of the GUP does not satisfy the property that the wave vector k asymptotically reaches the cutoff in large energy region as recently reported in Ref. [21].

Now, let us calculate the statistical entropy of a scalar field on the (1+1)-dimensional charged black hole background to all orders in the Planck length. When gravity is turned on, the number of quantum states in a volume element in phase cell space based on the GUP in the (1+1) dimensions is given by $dn_A = \frac{dr dp_r}{2\pi} e^{-\lambda p^2}$, where $p^2 = p^r p_r$ and one quantum

state corresponding to a cell of volume is changed from 2π into $2\pi e^{\lambda p^2}$ in the phase space [7, 8]. Then, the number of quantum states with energy less than ω is given by

$$\begin{aligned} n_A(\omega) &= \frac{1}{2\pi} \int dr dp_r e^{-\lambda p^2} \\ &= \frac{1}{\pi} \int dr \frac{1}{\sqrt{f}} \left(\frac{\omega^2}{f} - m^2 \right)^{\frac{1}{2}} e^{-\lambda(\frac{\omega^2}{f} - m^2)}. \end{aligned} \quad (15)$$

On the other hand, for the bosonic case the free energy at inverse temperature β is given by

$$F_A = \frac{1}{\beta} \sum_K \ln [1 - e^{-\beta \omega_K}], \quad (16)$$

where K represents the set of quantum numbers. By using Eq. (15), the free energy can be rewritten as

$$\begin{aligned} F_A &\approx \frac{1}{\beta} \int dn_A(\omega) \ln [1 - e^{-\beta \omega}] \\ &= - \int_{\mu\sqrt{f}}^{\infty} d\omega \frac{n_A(\omega)}{e^{\beta \omega} - 1} \\ &= - \frac{1}{\pi} \int_{r_+}^{r_+ + \epsilon} dr \frac{1}{f} \int_0^{\infty} d\omega \frac{\omega}{(e^{\beta \omega} - 1)} e^{-\lambda \frac{\omega^2}{f}}. \end{aligned} \quad (17)$$

Here, we have taken the continuum limit in the first line and integrated by parts in the second line. Furthermore, in the last line of Eq. (17), since $f \rightarrow 0$ near the event horizon, *i.e.*, in the range of $(r_+, r_+ + \epsilon)$, $\omega^2/f - \mu^2$ becomes ω^2/f although we do not require the little mass approximation.

Moreover, we are only interested in the contribution from the just vicinity near the horizon, $(r_+, r_+ + \epsilon)$, which corresponds to a proper distance of order in the minimal length, $\sqrt{e\lambda/2}$. This is because the entropy closes to the upper bound only in this vicinity, which it is just the vicinity neglected by the brick wall method [2, 3, 5]. Then, we have

$$\sqrt{\frac{e\lambda}{2}} = \int_{r_+}^{r_+ + \epsilon} \frac{dr}{\sqrt{f(r)}} \approx \sqrt{\frac{2\epsilon}{\kappa}}, \quad (18)$$

where κ is the surface gravity at the horizon of the black hole and it is identified as $\kappa = \frac{1}{2} \frac{df}{dr} |_{\beta=\beta_H} = 2\pi\beta_H^{-1}$. Note that the Taylor's expansion of $f(r)$ near the horizon is given by $f(r) \approx 2\kappa(r - r_+) + \mathcal{O}((r - r_+)^2)$.

Before calculating the entropy, let us mention that Yoon et. al. have recently suggested that since the minimal length $\sqrt{\lambda}$ in Eq. (18) is related to the brick wall cutoff ϵ , the integral

about r in the range of the near horizon should be carefully treated for a convergent entropy [13]. In particular, although the term $(e^{\beta\omega} - 1)$ in Eq. (17) with $x = \sqrt{\frac{\lambda}{f}}\omega$ was expanded in the previous works giving $\beta\sqrt{\frac{f}{\lambda}}x$, one may not simply expand up to the first order because since $0 \leq \frac{f}{\lambda} = \frac{2\kappa(r-r_+)}{\lambda} \leq \frac{2\kappa\epsilon}{\lambda} = \kappa^2$ near the horizon [23].

Now, let us carefully consider the integral about r near the horizon by extracting out the ϵ -factor through the Taylor's expansion of $f(r)$. Then, the free energy of F_A in Eq. (17) can be written as

$$F_A \approx -\frac{1}{\pi} \int_0^\infty d\omega \frac{\omega}{(e^{\beta\omega} - 1)} \Lambda_A(\omega, \epsilon), \quad (19)$$

where Λ_A is defined by

$$\Lambda_A \equiv \int_{r_+}^{r_++\epsilon} dr \frac{1}{2\kappa(r-r_+)} e^{-\frac{\lambda\omega^2}{2\kappa(r-r_+)}}. \quad (20)$$

By defining $t = \frac{\lambda\omega^2}{2\kappa(r-r_+)}$, $\Lambda_A(\omega, \epsilon)$ becomes

$$\Lambda_A = \frac{1}{2\kappa} \int_\xi^\infty dt \frac{1}{t} e^{-t} = \frac{1}{2\kappa} \Gamma(0, \xi), \quad (21)$$

where we have used the incomplete Γ -function given as $\Gamma(a, \xi) = \int_\xi^\infty t^{a-1} e^{-t} dt$ with $\xi \equiv \frac{\lambda\omega^2}{2\kappa\epsilon}$.

Then, the all order corrected entropy through the GUP is given by

$$S_A = \beta^2 \frac{\partial F_A}{\partial \beta} \big|_{\beta=\beta_H} \approx \left(\frac{16}{\pi^2} \delta \right) \frac{1}{2} > \frac{1}{4} (A_2) = \frac{1}{2}, \quad (22)$$

where the numerical value of δ with $y \equiv \beta\omega/2$ is given by

$$\delta = \int_0^\infty dy \frac{y^2}{\sinh^2 y} \Gamma\left(0, \frac{2y^2}{e\pi^2}\right) \approx 4.66, \quad (23)$$

and A_2 denotes the 2D area. This is the all order corrected finite entropy based on the GUP. We note that all order GUP corrected entropy does not give a logarithmic correction in the leading order WKB approximation [30].

Now, it seems appropriate to comment on the entropy (22), which is obtained through the Taylor expansion of $f(r)$ to all orders in the Planck length. Since $A_2 = 2$ from the relation of the area A_d of a sphere of radius r in d spatial dimensions as $A_d = 2\pi^{d/2} r^{d-1} / \Gamma(\frac{d}{2})$, we have failed to obtain the exact Bekenstein-Hawking entropy $S = \frac{1}{2}$ in contrast to the higher dimensional cases. The reason is that there is no adjustable parameter, which is actually the minimal length λ , in the two dimensions in contrast to the higher dimensional cases in which all order correction faction $\sqrt{\frac{e}{2}}$ contained in the enlarged minimal length can be absorbed by adjusting the parameter α as in Ref.[23, 24].

In summary, by using the generalized uncertainty principle, we have investigated the entropy to all orders in the Planck length of the massive scalar field within the just vicinity near the horizon of a static black hole in the (1+1)-dimensional charged black hole background by carefully considering the integral about r in the range $(r_+, r_+ + \epsilon)$ near the horizon without any artificial cutoff and little mass approximation and satisfying the asymptotic property of the wave vector k in the modified dispersion relation. In contrast to the previous results of the higher dimensional cases having adjustable parameter, we have obtained an unadjustable entropy due to the independence of the minimal length while the entropy is proportional to the Bekenstein-Hawking entropy.

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